

sample

total volume. Any more would overflow

15 mL of industrial effluent is mixed in a 300 mL BOD bottle. The initial DO of the mixture is 8 mg/L. After 5 days the DO concentration is 3 mg/L. What is the BOD at 5 days?

$$\text{BOD}_5 = \frac{\text{DO}_i - \text{DO}_f}{P} = \frac{8 \text{ mg/L} - 3 \text{ mg/L}}{15/300}$$

Unseeded

$$P = \frac{V_{\text{sample}}}{V_{\text{total}}}$$

Same P in both formulae

$$= 100 \text{ mg/L O}_2$$

A 300 mL bottle has 15 mL of wastewater. The DO drops 7.2 mg/L in 5 days. The blank has a DO drop of 1 mg/L in 5 days and a volume of seeded dilution water equal to 300 mL. What is the BOD at 5 days?

$$\text{BOD}_5 = \frac{(\text{DO}_i - \text{DO}_f) - (B_i - B_f)(1-P)}{P}$$

Seeded

$$P = \frac{15 \text{ mL}}{300 \text{ mL}} = 0.05$$

$$\Delta \text{DO} = -7.2 \text{ mg/L}$$

Drop → negative

Blank

$$\Delta B = -1 \text{ mg/L}$$

t = 5d

$$\begin{aligned} \text{BOD}_5 &= \frac{-\Delta \text{DO} - \Delta B (1-P)}{P} \\ &= \frac{+7.2 \text{ mg/L} - (-1 \text{ mg/L})(0.95)}{0.05} \\ &= 125 \text{ mg/L O}_2 \end{aligned}$$

$$\Delta = \text{final} - \text{initial}$$

Hints that you should use the seeded formula:
You're given a volume of **seeded** dilution water
You're given information about the **blank**

$$BOD_0 = 100 \text{ mg/L}$$

$$k = 0.1 \text{ d}^{-1}$$

If the BOD₁₀ of a waste is 100 mg/L and the BOD decay constant, k , is 0.1/d, what is the ultimate BOD?

 L_0

Days cancel

$$L_t = L_0 e^{-kt}$$

substitute

$$L_0 = BOD_t + L_t$$

to get

$$L_0 = BOD_t + L_0 e^{-kt}$$

→ Plug in and algebra. The t are the same

$$L_0 = 100 \text{ mg/L} + L_0 e^{-kt}$$

$$L_0 - L_0 e^{-1} = 100 \text{ mg/L}$$

Step I will often skip

$$L_0 (1 - e^{-1}) = 100 \text{ mg/L}$$

$$L_0 = \frac{100 \text{ mg/L}}{1 - e^{-1}} = 158 \text{ mg/L}$$

If the BOD₅ of a waste is 102 mg/L and the BOD₃₀ (corresponds to the ultimate BOD) is 158 mg/L, what is k (base e)?

$$L_0 = 158 \text{ mg/L}$$

$$BOD_5 = 102 \text{ mg/L}$$

 $t = 5$

Same steps to get this eq'n

$$L_0 = BOD_t + L_0 e^{-kt}$$

$$L_0 (1 - e^{-kt}) = BOD_t$$

$$158 \text{ mg/L} (1 - e^{-k(5)}) = 102 \text{ mg/L}$$

$$\ln(e^{-k(5)}) = \ln(1 - 0.65)$$

$$-k(5) = -1.04$$

$$k = 0.21 \text{ d}^{-1}$$

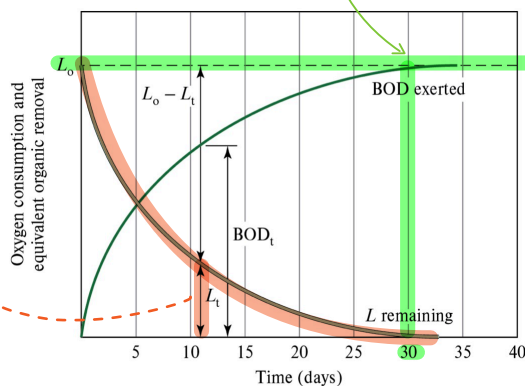


FIGURE 9-8

BOD and oxygen-equivalent relationships.

Course.	Title.	Date.
		Mo Tu We Th Fr Sa Su

Calculate the concentration of O₂ in water at equilibrium with air at 30°C.

I prefer atm/M units, which is why I provide that on exams, but this table uses atm per mole fraction, which I consider a more advanced version

TABLE A-11 Henry's Law Constants for Common Gases Soluble in H₂O

Temperature (°C)	$K_H \times 10^{-4}$ (atm)							
	Air	N ₂	O ₂	CO ₂	CO	H ₂	H ₂ S	CH ₄
0	4.32	5.29	2.55	0.073	3.52	5.79	0.027	2.2
10	5.49	6.68	3.27	0.104	4.42	6.36	0.037	2.9
20	6.64	8.04	4.01	0.142	5.36	6.83	0.048	3.7
30	7.71	9.24	4.75	0.186	6.20	7.29	0.061	4.4
40	8.70	10.4	5.35	0.233	6.96	7.51	0.075	5.2

Source: G. Kiely, *Environmental Engineering*. 1996 The McGraw-Hill Education Companies, New York.

14 IVA	15 VA	16 VIA
6 C Carbon 12.011 24	7 N Nitrogen 14.007 25	8 O Oxygen 15.999 26

assume the water is pure
Not true but a decent approximation

$$MW(O_2) = 16 \times 2 = 32$$

$$MW(H_2O) = 16 + 1.01 \times 2 = 18.02$$

$$P_{O_2} = P_{atm} (20.9\%) = 0.209 atm$$

1 atm
@ sea level

$$K_{H,O_2} = \frac{P_{O_2}}{\chi_{O_2}}$$

$$4.75 \times 10^{-4} = \frac{0.209}{\chi_{O_2}}$$

$$4.75 \times 10^{-4} \chi = 0.209$$

$$\chi = \frac{0.209}{4.75 \times 10^{-4}} = 4.4 \times 10^{-6} \frac{\text{moles } O_2}{\text{moles tot}}$$

since the mole fraction is so small, we can assume "total" is just H₂O

But we want this in mg/L to use in our calculations in this chapter

from table A-1 the density of water at 30°C is 995.650 kg/m³
(most of the time we can simplify to 1000 kg/m³ for this class)

$$\frac{4.4 \times 10^{-6} \text{ moles } O_2}{\text{moles } H_2O} \times \frac{32g O_2}{\text{mole } O_2} \times \frac{10^3}{18.02g H_2O} \times \frac{995.650kg}{m^3} \times \frac{1000mg}{g} = 7.78 \text{ mg/L}$$

$$= 7.78 \text{ mg/L}$$

the values in table A-2 are calculated more precisely. We will use these moving forward.

TABLE A-2 Saturation Values of Dissolved Oxygen in Fresh Water Exposed to a Saturated Atmosphere Containing 20.9% Oxygen Under a Pressure of 101.325 kPa^a

Temperature (°C)	Dissolved Oxygen (mg · L ⁻¹)	Saturated Vapor Pressure (kPa)
0	14.62	0.6108
1	14.23	0.6566
2	13.84	0.7055
3	13.48	0.7575
4	13.13	0.8129
5	12.80	0.8719
6	12.48	0.9347
7	12.17	1.0013
8	11.87	1.0722
9	11.58	1.1472
10	11.30	1.2264
11	11.03	1.3097
12	10.76	1.3972
13	10.50	1.4889
14	10.25	1.5848
15	10.00	1.6859
16	9.76	1.7921
17	9.52	1.9035
18	9.29	2.0200
19	9.06	2.1417
20	8.84	2.2686
21	8.62	2.4007
22	8.41	2.5380
23	8.20	2.6805
24	7.99	2.8283
25	7.79	2.9813
26	7.59	3.1395
27	7.39	3.3029
28	7.19	3.4715
29	6.99	3.6453
30	6.79	3.8244
31	6.59	4.0087
32	6.39	4.1983
33	6.19	4.3931
34	5.99	4.5932
35	5.79	4.7985
36	5.59	5.0091
37	5.39	5.2250
38	5.19	5.4462

Skip a few

atm, std

1.013×10^5

pascal (Pa)

so $101.3 \text{ kPa} = 1 \text{ atm}$

Source: Calculated by G. C. Whipple and M. C. Whipple from measurements of C. J. J. Fox, *Journal of the American Chemical Society*, vol. 33, p. 362, 1911.

A uniform natural stream, 2.2 m deep, flows with an average velocity of 0.4 m/s. The stream contains oxygen-demanding substances, which decay and utilize oxygen according to first-order kinetics. Just below the outfall from a wastewater, the ultimate BOD is 25 mg/L, the DO is 4.1 mg/L, and the temperature is 25°C. The $k_d = 0.31/\text{d}$ and $k_r = 0.76/\text{d}$, both determined at 25°C.

Determine

Subscript c

Initial ultimate BOD = L_a

A) the critical DO and critical distance

Table A-2 Saturation Values of Dissolved Oxygen in Fresh Water Exposed to a Saturated Atmosphere Containing 20.9% Oxygen Under a Pressure of 101.325 kPa^a

Temperature (°C)	Dissolved Oxygen (mg · L ⁻¹)	Saturated Vapor Pressure (kPa)
0	14.62	0.6108
25	8.38	3.1671

$$D_t = DO_{sat} - DO_t$$

$$DO_{sat}(25^\circ\text{C}) = 8.38 \text{ mg/L}$$

Definition of Deficit, general.

For initial deficit $t = 0$ or a to distinguish between 0 and a

$$D_a = 8.38 - 4.1 = 4.28 \text{ mg/L}$$

$$t_c = \frac{1}{k_r - k_d} \ln \left[\frac{k_r}{k_d} \left(1 - D_a \frac{k_r - k_d}{L_a} \right) \right] = \frac{1}{.76 - .31} \ln \left[\frac{.76}{.31} \left(1 - 4.28 \frac{.76 - .31}{25} \right) \right]$$

$$= 1.683 \text{ d}$$

$$k_r = 0.76 \text{ d}^{-1} \quad L_a = 25 \text{ mg/L}$$

$$k_d = 0.31 \text{ d}^{-1}$$

If you calculate this in steps, you might get something closer to 2 depending on rounding. If you want to calculate in steps, you might want to store the sub parts in your calculator.

This is the critical **time**. To get the critical ***distance*** remember that **velocity = distance/time**

$$\vec{v} = \frac{x_c}{t_c} \rightarrow x_c = 0.4 \text{ m/s} \left(\frac{1.683 \text{ days} | 24 \text{ h} | 3600 \text{ s}}{1 \text{ d} | 1 \text{ hr}} \right) = 58164 \text{ m} \text{ or } 58 \text{ km}$$

Critical Deficit: plug critical time into the deficit equation—this will give you the deficit at that time

$$D_c = \frac{k_d L_a}{k_r - k_d} \left(e^{-k_d t_c} - e^{-k_r t_c} \right) + D_a e^{-k_r t_c}$$

$$= \frac{(0.31) 25}{.76 - .31} \left(e^{-.31(1.683)} - e^{-.76(1.683)} \right) + 4.28 e^{-.76(1.683)} = 6.619 \text{ mg/L}$$

Critical DO

$$DO_{sat} - D_c = 8.38 \text{ mg/L} - 6.619 \text{ mg/L} = 1.76 \text{ mg/L}$$

B) the DO at 16 km downstream

$$\frac{16 \text{ km} | 1000 \text{ m}}{1 \text{ km}} \div \frac{0.4 \text{ m}}{1 \text{ s}} = \frac{16000 \text{ m}}{0.4 \text{ m/s}} = 40000 \text{ s} = \frac{40000 \text{ s} | 3600 \text{ s}}{1 \text{ hr} | 24 \text{ hr}} = 0.463 \text{ d} = t$$

$$D = \frac{k_d L_a}{k_r - k_d} \left(e^{-k_d t} - e^{-k_r t} \right) + D_a e^{-k_r t}$$

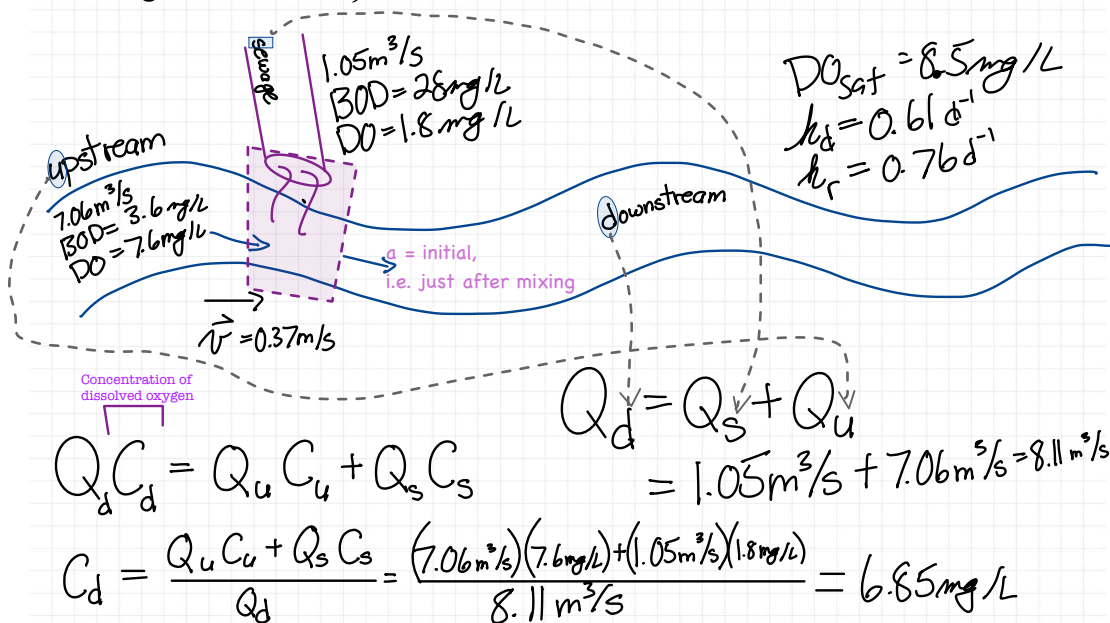
$$= \frac{(0.31) 25}{.76 - .31} \left(e^{-.31(0.463)} - e^{-.76(0.463)} \right) + 4.28 e^{-.76(0.463)} = 5.816 \text{ mg/L}$$

$$DO = DO_{sat} - D_t = 8.38 - 5.816 = 2.56 \text{ mg/L}$$

At the current temperature, the saturation DO in a river is 8mg/L. You measure DO and find that it is 6mg/L. What is D_t at that point?

$$D_t = DO_{sat} - DO_t \quad 8\text{mg/L} - 6\text{mg/L} = 2\text{mg/L}$$

A city of 200,000 people disposes of 1.05 m³/s of treated sewage that still has a BOD of 28 mg/L and DO of 1.8 mg/L into the river. Upstream from the outfall, the river has a flow rate of 7.06 m³/s and velocity of 0.37 m/s. At this point, the BOD and DO in the river are 3.6 and 7.6 mg/L, respectively. The saturation value of DO is 8.5 mg/L. The deoxygenating rate is 0.61/d and the re-aeration coefficient is 0.76/d. Assume complete mix and the velocity is the same up and downstream. What are the oxygen deficit and the BOD just downstream of the outfall (after mixing before reaction)? What is the DO 16km downstream?



You can convert units, but the m³ to L conversion will end up cancelling out since it would be in all the terms

$$D_a = DO_{sat} - DO = 8.5 \text{ mg/L} - 6.85 \text{ mg/L} = 1.65 \text{ mg/L}$$

We can follow a similar process to calculate the BOD. For the point just downstream of mixing, we are looking at initial ultimate BOD.

$$L_0 = \frac{3.6(7.06) + 28(1.05)}{7.06 + 1.05} = 6.76 \text{ mg/L}$$

$$\frac{16 \text{ km}}{\text{km}} \cdot \frac{1,000 \text{ m}}{\text{km}} \cdot \frac{\text{s}}{0.37 \text{ m}} \cdot \frac{\text{hr}}{3600 \text{ s}} \cdot \frac{\text{d}}{24 \text{ h}} = 0.5 \text{ d}$$

$\vec{v}$

$$D = \frac{k_d L_0}{k_r - k_d} (e^{-k_d t} - e^{-k_r t}) + D_a e^{-k_r t}$$

$$= \frac{0.61(6.76)}{0.76 - 0.61} (e^{-0.61(0.5)} - e^{-0.76(0.5)}) + 1.65 e^{-0.76(0.5)}$$

$$= 2.6 \text{ mg/l}$$

This is the deficit. To calculate the DO concentration, we need to subtract this from the saturation value

$$DO + D = DO_{\text{sat}}$$

$$DO = DO_{\text{sat}} - D$$

$$= 8.5 \text{ mg/l} - 2.6 \text{ mg/l} = 5.9 \text{ mg/l}$$