

Introduction to Environmental Engineering

Background knowledge, general engineering knowledge, and memory refresh

Important Mathematical tips and tricks to remember:

Exponent Rules

For nonzero real numbers a and b and integers (positive, whole numbers) m and n :

$$b^0 = 1$$

$$b^{-n} = \frac{1}{b^n}$$

$$\frac{1}{b^{-n}} = b^n$$

$$b^m b^n = b^{m+n}$$

$$\frac{b^m}{b^n} = b^{m-n}$$

$$(b^m)^n = b^{mn}$$

$$(ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$b^{1/n} = \sqrt[n]{b}$$

Log Rules

b , m , and n must all be positive.

Reminder that \ln is log base e ($b = e$).

The rules hold as long as b is > 1 , but typically $b = 10$ or e

$$\log_b(mn) = \log_b m + \log_b n$$

$$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$$

$$\log_b(m^k) = k \cdot \log_b m$$

$$\log_b(1) = 0$$

$$\log_b(b) = 1$$

$$\log_b(b^k) = k$$

$$b^{\log_b(k)} = k$$

Algebra insights

Multiply by "1", but use a creative definition of 1. Anything times one is the same value.

$$\text{ex } 1 = \frac{a}{a}$$

$$y = \frac{z}{a}$$

$$\left(\frac{a}{a}\right) y = \frac{z}{a}$$

$$\frac{ay}{a} = \frac{z}{a}$$

Multiply all terms by the same value

$$\left(\frac{ay}{a} = \frac{z}{a}\right) a \quad \begin{matrix} \uparrow \\ \text{distribute} \end{matrix}$$

$$ay = z$$

Add something to both sides.

$$y = mx - b$$

$$y + b = mx - b + b$$

$$y + b = mx$$

Combine like terms

Constants - a, b, c, d, e
variables/unknowns - x, y, z

$$ax^2 + by + c = bx + d + ey$$

$$(ax - b)x + (c - d) = (e - b)y$$

Sometimes you may want the x^2 and x separate

get all vars of same power w/ single coeff

Multiplying and dividing are interchangeable if you multiply by a fraction. Same for addition and subtraction with negatives.

$$\frac{ab}{c} \rightarrow \frac{ab}{c} \left(\frac{c}{c}\right) = ab$$

$$\frac{ab}{c} \rightarrow \frac{ab}{c} \left(\frac{1}{b}\right) = \frac{a}{c}$$

$$\alpha \rightarrow \alpha + \beta$$

$$\alpha \rightarrow \alpha + (-\beta) = \alpha - \beta$$

Distribute

$$a(\beta + \gamma) = a\beta + a\gamma$$

System of equations

Use the equations to define one variable in terms of another and reduce # of unknowns until you can solve

Quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

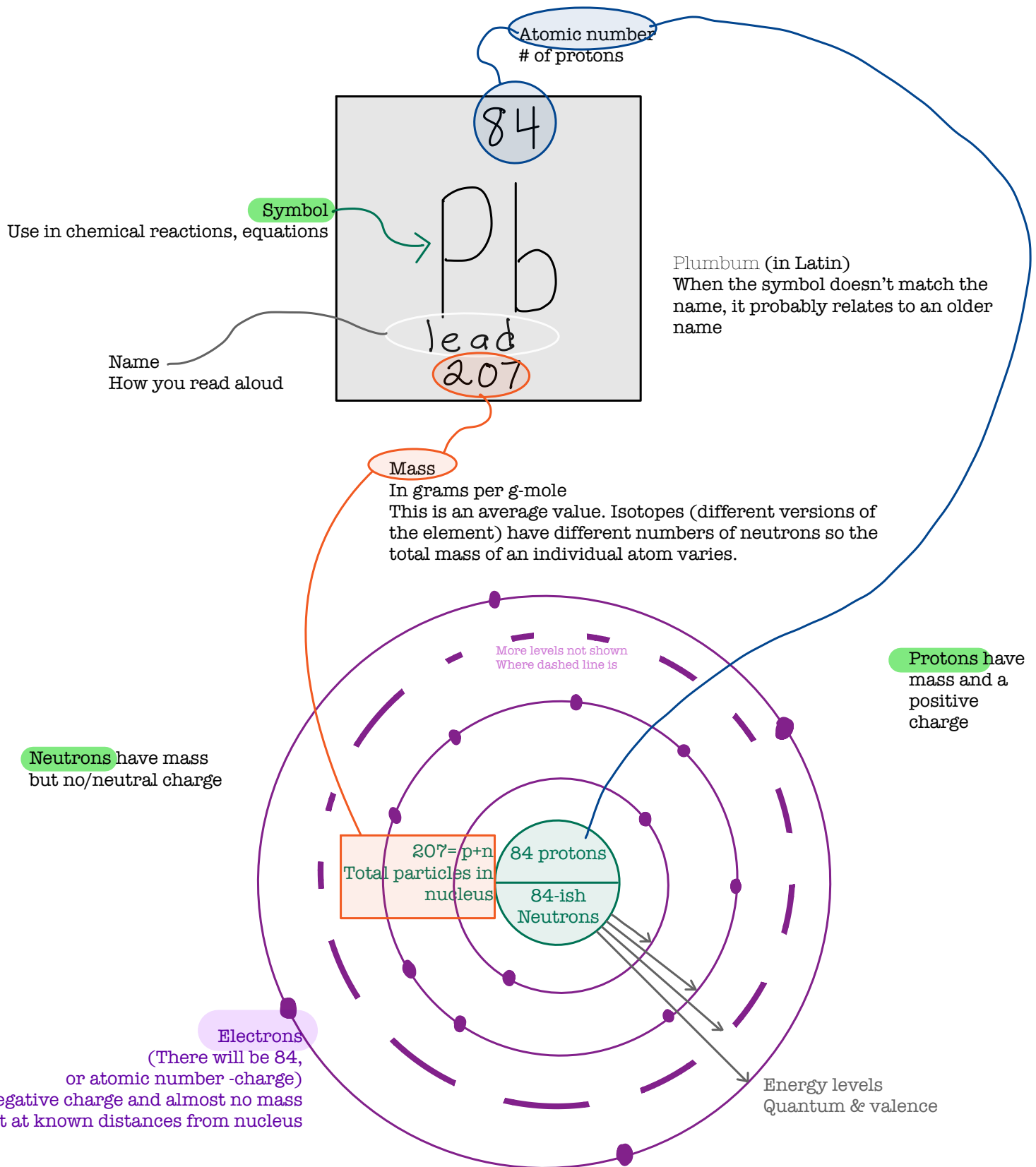
$$\text{when } ax^2 + bx + c = 0$$

The periodic table tells you about atoms

Atomic charge
mass
location in chart

tell you about the properties of the element and its interactions with other elements

Atomic properties are determined by (or determine) what element an atom is



Periodic Table

Organizes elements based on their subatomic structure, which defines their properties

cation

Positively charged
Donate an e-

anion

Negatively charged

Noble gasses
Not + or -
Unlikely to react
Inert

State of matter (color of name)
GAS LIQUID SOLID UNKNOWN

Subcategory in the metal-metalloid-nonmetal trend (color of background)
Alkali metals Lanthanides Metalloids
Alkaline earth metals Actinides Reactive nonmetals
Transition metals Post-transition metals Noble gases Unknown chemical properties

Atomic Number → 1
Symbol ← H
Name → Hydrogen
Atomic Weight ← 1.008
Electrons per shell → 1

1 IA H Hydrogen 1.008 1	2 IIA He Helium 4.0026 2											13 IIIA B Boron 10.81 2-3	14 IVA C Carbon 12.01 2-4	15 VA N Nitrogen 14.007 2-5	16 VIA O Oxygen 15.999 2-6	17 VIIA F Fluorine 18.998 2-7	18 VIIIA Ne Neon 20.180 2-8
3 Li Lithium 6.94 2-3	4 Be Beryllium 9.012 2-2	5 VB V Vanadium 50.942 2-8-2	6 VIB Cr Chromium 51.996 2-8-1-1	7 VIIB Mn Manganese 54.938 2-8-2-2	8 VIII Fe Iron 55.845 2-8-1-2	9 VIII Co Cobalt 58.933 2-8-1-2	10 VIII Ni Nickel 58.693 2-8-1-2	11 IB Cu Copper 63.546 2-8-1-1	12 IIB Zn Zinc 65.38 2-8-1-2	13 Al Aluminum 26.982 2-8-3	14 Si Silicon 28.085 2-8-4	15 P Phosphorus 30.974 2-8-5	16 S Sulfur 32.06 2-8-6	17 Cl Chlorine 35.45 2-8-7	18 Ar Argon 39.948 2-8-8		
19 K Potassium 39.098 2-8-1	20 Ca Calcium 40.078 2-8-2	21 Sc Scandium 44.956 2-8-2	22 Ti Titanium 47.867 2-8-2	23 V Vanadium 50.942 2-8-2	24 Cr Chromium 51.996 2-8-1-1	25 Mn Manganese 54.938 2-8-2-2	26 Fe Iron 55.845 2-8-1-2	27 Co Cobalt 58.933 2-8-1-2	28 Ni Nickel 58.693 2-8-1-2	29 Cu Copper 63.546 2-8-1-1	30 Zn Zinc 65.38 2-8-1-2	31 Ga Gallium 69.723 2-8-1-3	32 Ge Germanium 72.630 2-8-1-4	33 As Arsenic 74.922 2-8-1-5	34 Se Selenium 78.971 2-8-1-6	35 Br Bromine 79.904 2-8-1-7	36 Kr Krypton 83.798 2-8-1-8
37 Rb Rubidium 85.468 2-8-1-1	38 Sr Strontium 87.62 2-8-1-2	39 Y Yttrium 88.906 2-8-1-2	40 Zr Zirconium 91.224 2-8-1-2	41 Nb Niobium 92.906 2-8-1-2	42 Mo Molybdenum 95.94 2-8-1-3	43 Tc Technetium (98) 2-8-1-3	44 Ru Ruthenium 101.07 2-8-1-3	45 Rh Rhodium 102.91 2-8-1-3	46 Pd Palladium 106.42 2-8-1-3	47 Ag Silver 107.87 2-8-1-1	48 Cd Cadmium 112.41 2-8-1-2	49 In Indium 114.82 2-8-1-3	50 Sn Tin 118.71 2-8-1-4	51 Sb Antimony 121.76 2-8-1-5	52 Te Tellurium 127.60 2-8-1-6	53 I Iodine 126.90 2-8-1-7	54 Xe Xenon 131.29 2-8-1-8
55 Cs Cesium 132.905 2-8-1-1	56 Ba Barium 137.33 2-8-1-2	57-71 Lanthanides	72 Hf Hafnium 178.49 2-8-1-3	73 Ta Tantalum 180.948 2-8-1-3	74 W Tungsten 183.84 2-8-1-3	75 Re Rhenium 186.21 2-8-1-3	76 Os Osmium 190.23 2-8-1-4	77 Ir Iridium 192.22 2-8-1-3	78 Pt Platinum 195.08 2-8-1-3	79 Au Gold 196.97 2-8-1-1	80 Hg Mercury 200.59 2-8-1-2	81 Tl Thallium 204.38 2-8-1-3	82 Pb Lead 207.2 2-8-1-4	83 Bi Bismuth 208.98 2-8-1-4	84 Po Polonium (209) 2-8-1-4	85 At Astatine (210) 2-8-1-5	86 Rn Radon (222) 2-8-1-8
87 Fr Francium (223) 2-8-1-1	88 Ra Radium (226) 2-8-1-2	89-103 Actinides	104 Rf Rutherfordium (261) 2-8-1-3	105 Db Dubnium (262) 2-8-1-3	106 Sg Seaborgium (263) 2-8-1-3	107 Bh Bohrium (264) 2-8-1-3	108 Hs Hassium (265) 2-8-1-3	109 Mt Meitnerium (266) 2-8-1-3	110 Ds Darmstadtium (271) 2-8-1-3	111 Rg Roentgenium (272) 2-8-1-3	112 Cn Copernicium (285) 2-8-1-3	113 Nh Nihonium (286) 2-8-1-3	114 Fl Flerovium (289) 2-8-1-4	115 Mc Moscovium (290) 2-8-1-4	116 Lv Livermorium (293) 2-8-1-4	117 Ts Tennessine (294) 2-8-1-5	118 Og Oganesson (294) 2-8-1-8
57 La Lanthanum 138.91 2-8-1-1	58 Ce Cerium 140.12 2-8-1-2	59 Pr Praseodymium 140.91 2-8-1-2	60 Nd Neodymium 144.24 2-8-1-2	61 Pm Promethium (145) 2-8-1-2	62 Sm Samarium 150.36 2-8-1-2	63 Eu Europium 151.96 2-8-1-2	64 Gd Gadolinium 157.25 2-8-1-2	65 Tb Terbium 158.93 2-8-1-2	66 Dy Dysprosium 162.50 2-8-1-2	67 Ho Holmium 164.93 2-8-1-2	68 Er Erbium 167.26 2-8-1-2	69 Tm Thulium 168.93 2-8-1-2	70 Yb Ytterbium 173.05 2-8-1-2	71 Lu Lutetium 174.97 2-8-1-3			
89 Ac Actinium (227) 2-8-1-1	90 Th Thorium 232.04 2-8-1-2	91 Pa Protactinium 231.04 2-8-1-2	92 U Uranium 238.03 2-8-1-2	93 Np Neptunium (237) 2-8-1-2	94 Pu Plutonium (244) 2-8-1-2	95 Am Americium (243) 2-8-1-2	96 Cm Curium (247) 2-8-1-2	97 Bk Berkelium (247) 2-8-1-2	98 Cf Californium (251) 2-8-1-2	99 Es Einsteinium (252) 2-8-1-2	100 Fm Fermium (257) 2-8-1-2	101 Md Mendelevium (258) 2-8-1-2	102 No Nobelium (259) 2-8-1-2	103 Lr Lawrencium (260) 2-8-1-3			

Heavier as you descend

Each column will react (combine) similarly

- Commonly referenced elements with “weird” symbols & mnemonics
- Fe = iron (think iron man is Fe-male)
 - Ag = silver (if you win 2nd place you might think “ahh gee”)
 - Au = gold (if someone steals your gold, you’ll say “Hey you!”)
 - Hg = mercury (He’s a God)
 - Pb = lead (the lumps are leading you the same direction)

Molecular Mass (Sometimes called Molecular weight)

$$MW = \sum \text{atomic weights}$$

Periodic Table of the Elements

Number
Symbol
Name
Mass

Atomic mass is the **non-whole number**

$$MW(\text{CaCO}_3) = MW(\text{Ca}) + MW(\text{C}) + 3 \times (\text{O})$$

$$= 40.08 + 12.01 + 3 \times 16$$

48

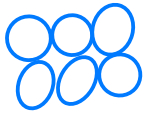
$$= 100.09 \text{ g/mol}$$

Phases of Matter

at a molecular level

solid

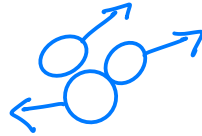
rigid



Relative molecular position constant

liquid

flowy



Constant distance between molecules
Relative positions change

gas

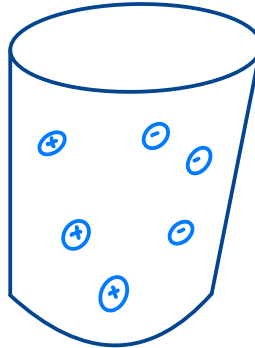
bouncy



Expands to fill container

solution * Technically not a phase

chemical solution



Solute dissociates to ions
I.e. changes at a molecular level

colloidal mixture



No chemistry
(small solids suspended in liquid)
Acts like a solution

Plasma - not used here

Dimensional Analysis

Problem: Your roommate is using your milk for their cereal every morning. They add $\frac{1}{4}$ cup of milk each morning, and you buy gallons of milk for \$4. During a 15 week semester, how much money have you lost due to stolen milk?

1. Create a "word bank" of fractions

$$\frac{0.25 \text{ c milk}}{\text{day}} \quad \frac{\$4}{\text{gallon}} \quad \frac{15 \text{ weeks}}{\text{semester}}$$

1.a) Look for any implicit fractions from standard conversion factors or stoichiometry

$$\frac{16 \text{ cups}}{\text{gallon}} \quad \frac{7 \text{ days}}{\text{week}}$$

2. Determine your starting and ending points

start \rightarrow already know $\rightarrow \frac{1/4 \text{ c}}{\text{day}}$
end \rightarrow what you want $\rightarrow \$/\text{semester}$

3. Fill in the middle to create a bridge from your starting point to your ending point

$\frac{1}{4} \text{ c milk}$	7 days	15 weeks	gallon	\$4
day	week	semester	16 cups	gallon

4. Multiply across the top and bottom *separately*. Then divide the product of the top by the product of the bottom. Now you're done!

$$\frac{0.25 \times 7 \times 15 \times 4}{16} = \frac{105}{16} = 6 \frac{9}{16}$$

$$\frac{\$6.56}{\text{Semester}}$$

The Greek Alphabet

In engineering, we often use the Greek alphabet for variables, so it's good to know which letters are which for discussing with others. In addition, many of the letters will often represent the same variable, in the same way that we typically use 'x' to represent the horizontal axis. I'm handwriting the letters so you can correlate my handwriting to the variables.

Name	Lower case	Upper case.	common use
Alpha	α	A	Common coefficient/unknown variable
Beta	β	B	reference, similar to x
Gamma	γ	Γ	We'll get to an atmospheric property that uses this
Delta	δ	Δ	Change (think derivatives)
Epsilon	ϵ	E	
Zeta	ζ	Z	
Eta	η	H	Efficiency
Theta	θ	Θ	Angles
Iota	ι	I	
Kappa	κ	K	Rate constants or other unit-less exponential terms
Lambda	λ (λ) <small>(Messier handwriting)</small>	Λ	Wavelengths of light
Mu	μ	M	Micro- (SI prefix)
Nu	ν	N	Light, photon energy
Xi	ξ	Ξ	
Omicron	\omicron	O	
Pi	π	Π	3.14159.....
Rho	ρ	P	Density
Sigma	σ	Σ	LC is frequently used, but common definition varies by field. UC represents summation
Tau	τ	T	
Upsilon	υ	Υ	
Phi	ϕ	Φ	
Chi	χ	χ	Sometimes refers to an unknown function, not a single variable, e.g. X(x)
Psi	ψ	Ψ	
Omega	ω	Ω	

Handwriting

Here's how I write the Latin (English) alphabet as well, in case you are finding something difficult to decipher

a b c d e f g h i j k l m n o p q r s t u v w x y z
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

f Sometimes I use cursive f's by themselves
a b c d e *f* g h i j k l m n o p q r s t u v w x y z

And here's how I write a few common mathematical symbols

1 2 3 4 5 6 7 8 9 0 [0] Sometimes I'll strike through a zero
if it's likely to be confused for an oh

Plus
+

minus
-

divide
 \div $\sqrt{\quad}$

multiply.
 \cdot \times

Parenthesis.
() []

And
 $\&$

$\sqrt{\quad}$

$\sqrt[3]{\quad}$